The Ultra-Violet Catastrophe

Any object that has been heated emits radiation. Human beings emit infra-red radiation. The filament in a light bulb emits a spectrum of radiation ranging from the infra-red to visible light. The sun emits a spectrum of radiation ranging from the infra-red to ultra-violet and beyond. In the 1890's physicists faced a paradox in trying to explain their observations of how bodies emit radiation. Classical physics made two assumptions. The first was that light is a wave, a continuous stream of energy flowing from an atom. The second was that, when an object emits light, the frequency of that light should be chosen at random. Unfortunately, these two assumptions led to what was called the "Ultra-Violet Catastrophe".

Radio waves, microwaves, infra-red, visible light, ultra-violet, X-rays and gamma rays are all forms of electromagnetic radiation. The frequencies associated with infra-red run from approximately 10^{11} Hz to 4×10^{14} Hz. Subtracting these two values leaves us about 4×10^{14} integer values for the frequency of infra-red light. The frequencies associated with ultra-violet radiation run from approximately 8×10^{14} Hz to 3×10^{17} Hz. Subtracting these two values leaves us about 3×10^{17} integer values for the frequency of ultra-violet light.

The frequency of the emitted light does not have to be an integer, of course. But we can see that there are about 750 times as many frequencies in the ultra-violet range as there are in the infra-red. This suggests that, if the frequency is chosen at random, there should be approximately 750 times more ultra-violet radiation emitted than infra-red. The problem becomes even worse if we consider the vast range of frequencies beyond the ultra-violet in the realm of X-rays and gamma-rays. According to classical physics, any heated object should be bathing us with large (and lethal) doses of ultra-violet, X-rays, and gamma-rays.

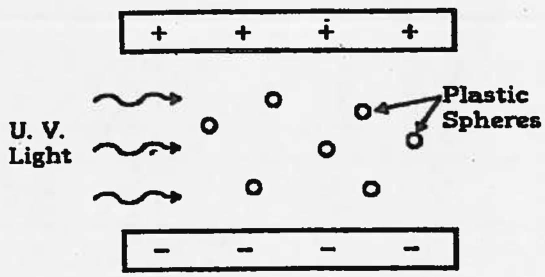
The important thing about the "Ultra-Violet Catastrophe" is that it doesn't happen. The problem lies in explaining why lower frequencies of light are much more likely to be emitted than higher frequencies. In 1900, Max Planck proposed a solution to this problem. He suggested that atoms do not emit light smoothly and continuously. Instead, energy is emitted from an atom in a bundle or packet which he called a quantum.

Planck also calculated that the energy possessed by a quantum is hf, where f is the frequency of the emitted light and h is a constant, called Planck's constant, with a value of 6.63×10^{-34} J.s. Violet light is emitted in quanta twice the size of those of red light since its frequency is twice as large. An atom cannot emit violet light until it has accumulated enough energy to make up a full quantum. However, the probability is that the atom will emit a quantum of lower frequency light before it has stored up the energy required for a quantum of violet light. This explains why light is radiated chiefly at low frequencies.

As the temperature of an object rises, the increased amount of energy available makes it more likely that an atom will have time to gain enough energy to emit quanta of high frequency light. Thus a kettle full of boiling water, at 100° C, emits all of its radiation in the infra-red spectrum. The surface of the sun, at 6 000° C, emits more radiation in the visible spectrum than at any other frequency.

Planck's explanation of the "Ultra-Violet Catastrophe" required a radical departure from the concepts of classical physics. Most physicists ignored his theory. Even Planck himself was dubious and tried to find an alternative explanation. However, a clerk in a Swiss patent office named Albert Einstein read about Planck's quantum theory and made use of it in 1905.

The Particle-like Properties of Light



In an experiment, tiny charged plastic spheres are held in balance between two horizontal charged plates. Light from an ultra-violet lamp falls on the spheres. The ultra-violet light can supply energy to the electrons on the surface of the spheres. An electron that gains energy will be able to escape from the sphere that it is on. As a result, that sphere will lose charge; the amount of electric force will decrease and the sphere will start to fall. If light is wave-like in nature, it is reasonable to expect that all of the spheres would start to fall at approximately the same time.

There might be a delay between the time that the lamp is turned on and the time that the spheres begin to fall. This time lag would be similar to that observed when sound waves are used to make a piece of crystal vibrate. The sound waves keep adding energy to the crystal until it shatters. Waves of ultra-violet light would keep adding energy to the electrons until they have enough to escape. As with sound waves, increasing the intensity (brightness) of the ultra-violet light should result in less of a time lag.

When this experiment is performed, however, the following observations are made:

1. Instead of a definite time lag during which energy is building up, it is observed that one or two spheres began to move immediately, whereas others remain stationary for a long time.
2. The amount of time between the lamp being turned on and the time when a particular sphere begins to fall appears to be strictly random.
3. The time lag does not appear to depend on the intensity of the ultra-violet light.

Photons

To explain these observations, it is assumed that light energy does not come in a continuous stream, but appears in small packets or bundles of energy called **photons**. These photons eject electrons at random. The energy of a photon depends on the frequency of the light. Light energy is quantized, that is, it comes in discrete (fixed) amounts.

$$E_{ph} = h f$$

E_{ph} = energy of a photon in J or eV

h = Planck's constant

$= 6.63 \times 10^{-34}$ J.s $= 4.14 \times 10^{-15}$ eV.s

f = frequency in Hz

The quantity hf is called a quantum of energy.

From the wave equation

$$c = f\lambda$$

$$E_{ph} = \frac{hc}{\lambda}$$

c = speed of light

= 3.0×10^8 m/s

λ = wavelength in m

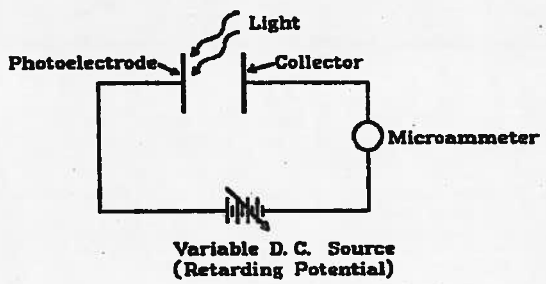
Wavelength is sometimes measured in angstrom (A) units and nanometres.

$$1\text{ A} = 10^{-10}\text{ m} \qquad 1\text{ nm} = 10^{-9}\text{ m}$$

The Photoelectric Effect

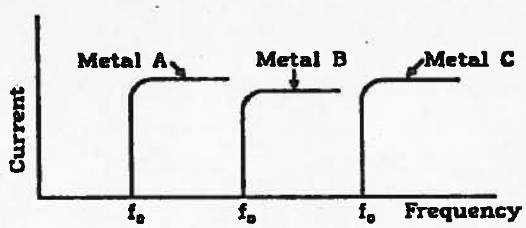
When light strikes a metal plate, electrons may be emitted from the plate and move away with a definite amount of kinetic energy. **The emission of electrons from certain materials by light of certain frequencies is called the Photoelectric Effect.**

This effect may be demonstrated using the apparatus below.



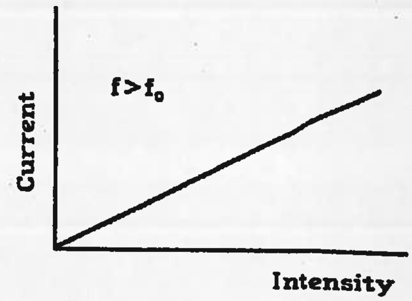
The following observations were made:

1. A photocurrent is observed when the light is above a certain frequency called the threshold frequency (f_o). **The threshold frequency is the minimum frequency at which electrons can be ejected from a photoelectric material.** The threshold frequency depends on the type of material in the photoelectrode.

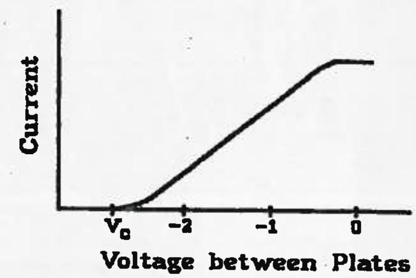


2. If the frequency is less than the threshold frequency, no electrons are emitted from the photoelectric surface regardless of the intensity (brightness) of the light.

3. If the frequency is greater than the threshold frequency, a photocurrent appears immediately when the light is turned on. As the light intensity increases, the amount of photocurrent increases.



4. As the retarding potential is increased and the collector plate becomes more negative, the photocurrent decreases. For some "cut-off potential", V_c, the photocurrent becomes zero. All electrons emitted at the photoelectrode are turned back by the retarding potential before they reach the collector plate.

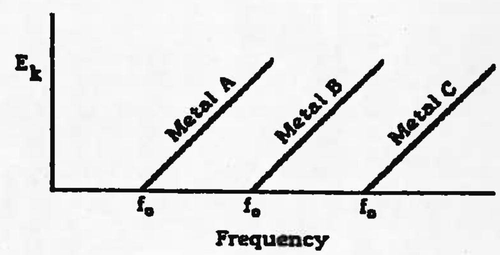


5. If light of the same frequency (greater than f_o) falls on a photoelectric surface, it is found that the cut-off voltage (V_c) remains the same regardless of the intensity of the light.

6. For frequencies of light, greater than the threshold frequency, it is found that the cut-off potential increases as the frequency increases for the same photoelectric surface. The maximum kinetic energy of the emitted photoelectrons can be calculated from the cut-off potential.

$E_k = E_e = qV_c$ $\qquad$ E_e = electric potential energy

7. If different photoelectric surfaces are exposed to light of various frequencies, the graphs of the kinetic energy for the ejected electrons versus the frequency is a set of parallel lines with the same slope.



In 1905 Albert Einstein explained the Photoelectric Effect using the following ideas:

1. When exposed to light, an electron from an atom of the photoelectrode accepts the entire energy of the incident photon, which thereby annihilates itself. The energy of a photon can be calculated using the following:

$$E_{ph} = hf$$

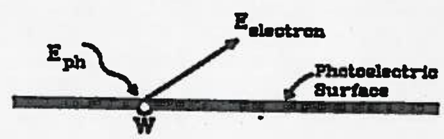
E_{ph} = energy of a photon in J or eV
h = Planck's constant
= 6.63×10^{-34} J.s = 4.14×10^{-15} eV.s
f = frequency in Hz

2. The energy gained by an electron might be great enough for it to escape from the surface of the photoelectrode. **The amount of energy required to just free the electron at the threshold frequency (f_o) is called the work function (W) of the photoelectrode.** The work function is sometimes called the binding energy or the threshold energy and can be calculated using the following:

$$W = hf_o$$

W = work function in J or eV
f_o = threshold frequency in Hz

3. Any excess energy donated to the electron, over and above the work function, appears as kinetic energy of the liberated electron.

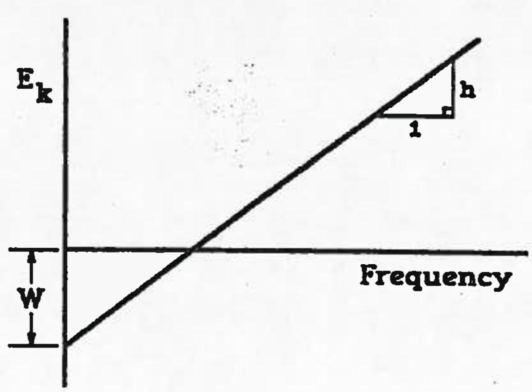


$$E_{ph} = W + E_k$$

$$hf = hf_o + \tfrac{1}{2}m_e v^2$$

m_e = mass of the electron

The Graph of Kinetic Energy versus Frequency



For any substance the graph of E_k vs f is a straight line.

Slope = h (same for all materials)

Intercept = W (this depends on the type of material)

The equation of the straight line is

$$E_k = hf - W$$

The Mechanics of the Ejected Electrons

For the emitted electrons, whose speed is not near the speed of light, the following equations are applicable:

$$E_k = \frac{1}{2} m_e v^2 \quad \text{.............. (i)}$$

$$p = m_e v$$

Multiply both sides of equation (i) by m_e

$$m_e E_k = \frac{1}{2} (m_e v)^2$$

Substitute for p

$$m_e E_k = \frac{1}{2} p^2$$

$$p^2 = 2 m_e E_k$$

$$p = \sqrt{2 m_e E_k}$$

E_k = kinetic energy in J
m_e = mass of the electron in kg
v = speed of the electron in m/s
p = magnitude of the momentum of the electron in kg.m/s

The Mechanics of the Photon

For photons, which travel at the speed of light, the above formulae no longer hold.

$$E_{ph} = hf$$

$$c = f\lambda$$

$$E_{ph} = \frac{hc}{\lambda}$$

E_{ph} = energy of a photon in J or eV
h = 6.63×10^{-34} J.s = 4.14×10^{-15} eV.s
f = frequency in Hz
c = 3.0×10^8 m/s
λ = wavelength

Consider Einstein's equation, $E = mc^2$, which suggests that mass can be converted into energy.

Hence the mass equivalence of a body with energy E is $\frac{E}{c^2}$

Momentum of a photon = Mass equivalence x speed of light

$$p = \left(\frac{E_{ph}}{c^2}\right) c$$

$$p = \frac{E_{ph}}{c}$$

$$p = \frac{hf}{c}$$

$$p = \frac{h}{\lambda}$$

p = magnitude of the momentum of the photon in kg.m/s
E_{ph} = energy of a photon in J
c = 3.0×10^8 m/s
f = frequency in Hz
λ = wavelength in m

Note: (i) A wavelength as well as a frequency is associated with the photon.
(ii) When a photon collides with an atom energy is transferred and momentum is transferred.

Experimental Verification

In 1916, Robert Millikan completed a series of experiments in which he measured the energy of the electrons emitted by light of different frequency. His observations agreed completely with Einstein's predictions. As a result, the quantum theory was universally accepted by physicists. In 1917, Albert Einstein was awarded the Nobel Prize in Physics for his 1905 explanation of the Photoelectric Effect.

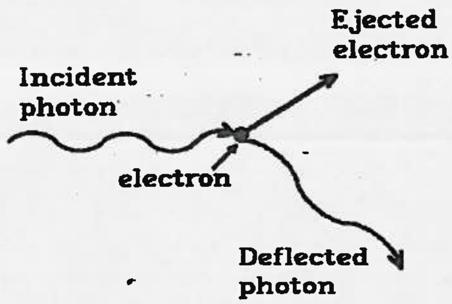
Applications of the Photoelectric Effect

- Burglar alarms and automatic door openers make use of a photocell circuit and a beam of light. When the beam of light is broken, the current suddenly drops to zero and this activates a switch such as a solenoid which controls the alarm or door.
- Some smoke detectors utilize the photoelectric effect. The presence of smoke blocks some of the light falling on a photoelectric surface. As a result, the amount of electric current in the circuit changes, thereby activating the detector.
- Photographic light meters also use a photocell circuit. As the light intensity increases, the current also increases.
- The frequencies on the soundtrack of films is detected by a photocell which receives the "modulated" light as it passes through the narrow shaded section at the side of the film.
- Calculators equipped with photocells do not require a battery. They can function anywhere if enough light falls on the photoelectric surface.

Photographic Film and the Quantum Nature of Light

Photographers in Planck's time knew that their film was very sensitive to the violet end of the spectrum and rather insensitive to the red end. A light bulb capable of producing only infra-red and red light could be used in a "darkroom" where film was developed. Using Planck's hypothesis about the nature of emitted light, it is easy to understand why this is so. A photographic film is covered with silver nitrate. Where suitable light strikes the film, a chemical reaction takes place and an image is formed. Obviously, the large quanta of violet light can produce a chemical reaction with much greater ease than the small quanta of red light.

The Compton Effect



A.H. Compton performed experiments in which a thin metal foil was bombarded with high energy photons (actually X-rays) with a short wavelength. The metal used had a very small work function. As a result, the incident photons were not absorbed. Instead an elastic collision took place in which electrons were ejected from the metal foil and the photons bounced off with lower energy. The photons after the collision had a lower frequency (or greater wavelength) than the incident photons. (Actually, very little energy is transferred to the electron. In some cases the electron may not even escape from the surface of the metal foil.)

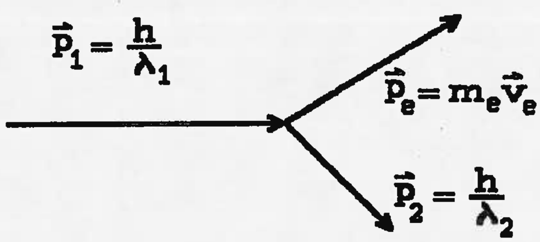
In this interaction energy is conserved.

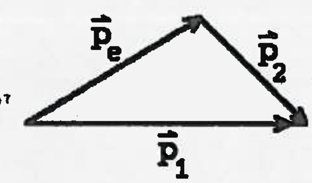
$$E_1 = E_2 + E_k$$

$$hf_1 = hf_2 + \frac{1}{2}m_e v_e^2$$

E_1 = energy of the incident photon
E_2 = energy of the deflected photon
E_k = kinetic energy of the electron
h = Planck's constant
f_1 = frequency of the incident photon
f_2 = frequency of the deflected photon
m_e = mass of the electron
v_e = speed of the electron

Momentum is also conserved in this type of collision. It was Compton who derived the expression for the momentum of a photon ($\vec{p} = \frac{h}{\lambda}$) using Einstein's equation $E = mc^2$.





$\vec{p}_1$ = momentum of the incident photon

$\vec{p}_2$ = momentum of the deflected photon

$\vec{p}_e$ = momentum of the electron after collision

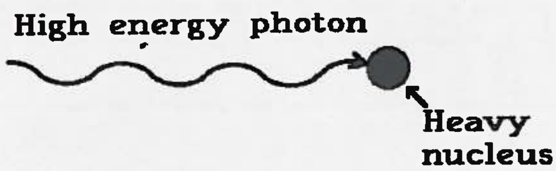
λ_1 = wavelength of the incident photon

λ_2 = wavelength of the deflected photon

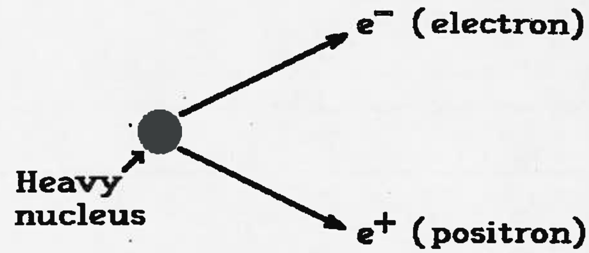
The experiments performed by Compton show the particle-like properties of light. Just like particles, photons possess both energy and momentum.

Pair Production

When a high energy photon (gamma rays and X-rays) collides with a heavy nucleus, it is destroyed and an electron and a positron are formed. A positron has the same mass as an electron but its charge is positive. Here matter is created from the energy of the photon in accordance with Einstein's equation ($E = mc^2$). This process in which a photon becomes an electron-positron pair is called pair production.



Before Pair Production



After Pair Production

Pair Annihilation

When a positron collides with an electron they destroy each other and produce photons. In this case matter is transformed into energy ($E = mc^2$).

Energy of Photons

Light energy appears in small bundles called photons.

$$E_{ph} = hf$$

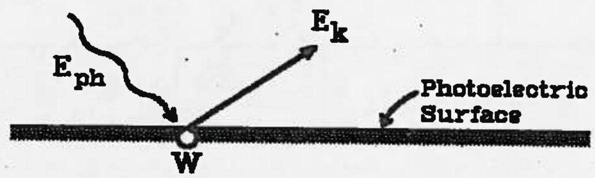
$$c = f\lambda$$

$$E_{ph} = \frac{hc}{\lambda}$$

E_{ph} = energy of a photon (J)
f = frequency (Hz)
c = 3.0×10^8 m/s
λ = wavelength (m)
h = Planck's constant
= 6.63×10^{-34} J.s
= 4.14×10^{-15} eV.s

The Photoelectric Effect

When a photoelectric surface is exposed to light and the energy of the bombarding photons is greater than the work function (binding energy), electrons are ejected from the surface.



$$E_{ph} = W + E_k$$

$$hf = hf_o + \frac{1}{2}m_e v_e^2$$

E_{ph} = energy of a photon (J)
W = work function (binding energy) (J)
E_k = kinetic energy of an electron (J)
f_o = threshold frequency (Hz)
m_e = mass of an electron (kg)
v_e = speed of ejected electron (m/s)

Momentum of a Photon

$$p = \frac{E_{ph}}{c}$$

$$p = \frac{hf}{c}$$

$$p = \frac{h}{\lambda}$$

p = magnitude of the momentum (kg.m/s)

Note: A photon has a wavelength as well as a frequency.
A photon has no mass.

Momentum of a Particle with Mass

$$p = mv$$

$$p = \sqrt{2mE_k}$$

p = magnitude of the momentum (kg.m/s)
m = mass of particle (kg)
v = speed of particle (m/s)
E_k = kinetic energy (J)